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$\therefore\ v+w=2r^2\pm2r\sqrt{4R^2+r^2}$ ....(14).

$(14)^2-4(7)$, etc. $v-w=2r[4R^2+2r^2-4x^2\pm2r\sqrt{4R^2+r^2}]^{\frac{1}{2}}$ ....(15).

$(14)\pm(15)$ after substituting the given values, $v=12288$, and $w=9408$.

(1), (5), and (6) $x=112$, $y=56$, and $z=16$, and the required sides AB, BC, CD, and AD are 168 rods, 128 rods, 56 rods, and 96 rods, respectively.

Also $BD=158.22$ rods.

Also solved by *CHARLES C. CROSS* and *H. C. WILKES.*

CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

61. Proposed by W. H. CARTER, Professor of Mathematics, Centenary College of Louisiana, Jackson, La.

If $r=a\sin n\theta$ is the polar equation of a curve, show (1) that the curve consists of n or $2n$ loops according as n is an odd or an even integer; (2) that its area is ¼ or ½ of the circumscribing circle according as n is an odd or an even integer.

I. Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics in Russell College, Lebanon, Va.

$r=a\sin n\theta$. Let $r=0$, then $\sin n\theta=0$.

$\therefore\ \theta=0,\ 2\pi/n,\ 4\pi/n,\ 6\pi/n,\ 8\pi/n,\ \ldots\ldots$, are the angles at which the the curve cuts the polar axis at the pole.

$dr/d\theta=na\cos n\theta=0$. $\therefore\ \theta=\pi/2n,\ 3\pi/2n,\ 5\pi/2n,\ 7\pi/2n,\ \ldots\ldots$, gives the points where r has its greatest value, namely, $\pm a$.

When n is odd the values of $n\theta$ for the angles $0,\ 2\pi/n,\ 4\pi/n,\ 6\pi/n,\ 8\pi/n,\ \ldots\ldots$, are $0,\ 2\pi,\ 4\pi,\ 6\pi,\ 8\pi,\ \ldots\ldots$

When n is even the values of $n\theta$ for the angles $0,\ 2\pi/n,\ 4\pi/n,\ 6\pi/n,\ 8\pi/n,\ \ldots\ldots$, are $0,\ \pi,\ 2\pi,\ 3\pi,\ 4\pi,\ \ldots\ldots$

$\therefore$ When n is even the polar axis is cut, at the pole, $2n$ times, but only n times when n is odd.

A=area of one loop.

$$A=\tfrac{1}{2}a^2\int_0^{\pi/n}\sin^2 n\theta d\theta,\ =\frac{\pi a^2}{4n}.$$

$$\therefore\ \frac{\pi a^2}{4n}\times n=\frac{\pi a^2}{4},\ \text{for } n \text{ odd};\ \frac{\pi a^2}{4n}\times 2n=\frac{\pi a^2}{2},\ \text{for } n \text{ even}.$$

II. Solution by E. L. SHERWOOD, A. M., Superintendent of City Schools, West Point, Miss.

Equation given $\rho=a\sin n\theta$. We may oberve that,

$$\rho=0,\ a,\ 0,\ -a, \text{ etc., when}$$

$$\sin n\theta=0,\ 1,\ 0,\ -1, \text{ etc., when}$$

$$n\theta=0,\ \tfrac{1}{2}\pi,\ \pi,\ \tfrac{3}{2}\pi, \text{ etc., when}$$

$\theta=\{[c/n].\frac{1}{2}\pi\}$, where c is 0, 1, 2, 3, 4, etc., up to $4n$ ($4n$ being determined by $\theta=2\pi$).

The series of values will be as follows:

$$\theta=0.\frac{\pi}{2n},\quad \frac{\pi}{2n},\ 2.\frac{\pi}{2n},\quad 3\frac{\pi}{2n}\ldots\ldots n.\frac{\pi}{2n},\quad [n+1]\frac{\pi}{2n}\ \ldots\ldots 2n\frac{\pi}{2n},$$

$$[2n+1]\frac{\pi}{2n}\ \ldots\ldots\ ;$$

If n is *even*, $\rho=0,\ a,\ 0,\ -a\ldots\ldots 0\pm a\ldots\ldots 0,\ a$.
If n is *odd*, $\rho=0,\ a,\ 0,\ -a\ldots\ldots\pm a,\ 0\ldots\ldots 0,\ -a\ldots\ldots$

$$\left\{\begin{array}{cccc} 3\,n\,.\frac{\pi}{2n}, & [3n+1]\frac{\pi}{2n} & \ldots\ldots\ldots\ldots & 4n.\frac{\pi}{2n} \\ 0 & \pm a & \ldots\ldots\ldots\ldots & 0 \\ \pm a & 0 & \ldots\ldots\ldots\ldots & 0 \end{array}\right.$$

In each series are $4n$ terms (the first coincides with the last), and $\rho=a$ numerically in $2n$ of them. But when n is odd, the radius vector traces each loop twice for $\pi/2n$ and a is the same point as $\{[2n+1][\pi/2n]\}$ and $-a$.

$\therefore$ There are $2n$ loops when n is even, and n loops when n is odd.

$$\text{Area } =\tfrac{1}{2}\int\rho^2d\theta, \text{ where } \rho^2=a^2\sin^2n\theta,$$

$$=\tfrac{1}{2}a^2\int\sin^2nd\theta,$$

$$=\tfrac{1}{2}a^2\left[\tfrac{1}{2}\theta-\frac{\sin2n\theta}{4n}\right]_0^{\pi/2n}=\frac{\pi a^2}{8n} \text{ for } \tfrac{1}{2} \text{ loop},$$

or $\pi a^2/4n$ for an entire loop.

$\therefore$ For $2n$ loops, area$=\pi a^2/2$; and for n loops, area$=\pi a^2/4$.

Also solved by *J. SCHEFFER* and *C. W. M. BLACK*.

62. Proposed by A. H. HOLMES, Brunswick, Maine.

A bucket is in the form of a frustum of a cone having its smaller end as a base. It is a inches in diameter at base and b inches in diameter at top, and its perpendicular height is c inches. It contains water the perpendicular height of which is $\frac{1}{2}c$ inches. What is the greatest height, from the plane on which the vessel rests, to which the surface of the water will rise when the bucket is overturned, no allowance being made for the thickness of the material of the bucket?